tinct from *the feather*, *the eagle*, and others, with which they have been hitherto confounded, and which he represents by the Hebrew Aleph.

The Rev. Charles Graves, F.T.C.D., read a paper "On the Application of Analysis to spherical Geometry."

The object of this paper is to investigate and apply to the geometry of the sphere, a method strictly analogous to that of rectilinear coordinates employed in plane geometry.

Through a point O on the surface of the sphere, which is called *the origin*, let two fixed quadrantal arcs of great circles OX, OY, be drawn; then if arcs be drawn from Y and X through any point P on the sphere, and respectively meeting OX and OY in M and N, the trigonometric tangents of the arcs OM, ON, are to be considered as the *coordinates* of the point P, and denoted by x and y. The fixed arcs may be called *arcs of reference*. An equation of the first degree between x and y represents a great circle; an equation of the second degree, a spherical conic; and, in general, an equation of the n^{th} degree, between the spherical coordinates x and y, represents a curve formed by the intersection of the sphere with a cone of the n^{th} degree, having its vertex at the centre of the sphere.

Though it is not easy to establish the general formulæ for the transformation of spherical coordinates, they are found to be simple.

Let x and y be the coordinates of a point referred to two given arcs, and let x', y', be the coordinates of the same point referred to two new arcs, whose equations as referred to the given arcs are

$$y - y'' = m\,(x - x''),$$
$$y - y'' = m'\,(x - x''),$$

x'', y'', being the coordinates of the new origin; then the values of x and y to be used in the transformation of coordinates would be

$$x = \frac{x''(ax' + by' - 1)}{px' + qy' - 1},$$

$$y = \frac{y''(cx' + dy' - 1)}{px' + qy' - 1}.$$

In which a, b, c, d, p, and q, are functions of m, m', x'', and y''. It is evident that the degree of the transformed equation in x', y', will be the same as that of the original one in x and y.

The great circle represented by the equation

$$\alpha x + \beta y = 1,$$

meets the arcs of reference in two points, the cotangents of whose distances from the origin are α and β; and, if the arcs of reference meet at right angles, the coordinates of the pole of this great circle are $-\alpha$, and $-\beta$. It appears from this, that if α and β, instead of being fixed, are connected by an equation of the first degree, the great circle will turn round a fixed point. And, in general, if α and β be connected by an equation of the n^{th} degree, the great circle will envelope a spherical curve to which n tangent arcs may be drawn from the same point. Thus, the fundamental principles of the theory of polar reciprocals present themselves to us in the most obvious manner as we enter upon the analytic geometry of the sphere.

A spherical curve being represented by an equation between rectangular coordinates, the equation of the great circle touching it at the point x', y', is

$$(y - y')\,dx' - (x - x')\,dy' = 0\,;$$

the equation of the normal arc at the same point is

$$\begin{aligned}(y - y')\,[dy' + x'(x'dy' - y'dx')]\\ + (x - x')\,[dx' + y'(y'dx' - x'dy')] = 0.\end{aligned}$$

Now, if we differentiate this last equation with respect to

x' and y', supposing x and y to be constant, we should find another equation, which, taken along with that of the normal arc, would furnish the values of x and y, the coordinates of the point in which two consecutive normal arcs intersect: and thus, as in plane geometry, we find the evolute of a spherical curve.

Let 2γ be the diametral arc of the circle of the sphere which osculates a spherical curve at the point x', y', Mr. Graves finds that

$$\tan\gamma = \pm \frac{[dx'^2 + dy'^2 + (x'dy' - y'dx')^2]^{\frac{3}{2}}}{(1 + x'^2 + y'^2)^{\frac{3}{2}}(dx'd^2y' - dy'd^2x')}.$$

For the rectification and quadrature of a spherical curve given by an equation between rectangular coordinates, the following formulæ are to be employed:—

$$ds = \frac{\sqrt{dx'^2 + dy'^2 + (x'dy' - y'dx')^2}}{1 + x'^2 + y'^2},$$

and

$$d\,(\text{area}) = \frac{ydx}{(1 + x^2)\sqrt{1 + x^2 + y^2}}.$$

In the preceding equations the radius of the sphere has been supposed $= 1$.

The method of coordinates here employed by Mr. Graves is entirely distinct from that which is developed by Mr. Davies in a paper in the 12th Vol. of the Transactions of the Royal Society of Edinburgh. Mr. Graves apprehends, however, that he has been anticipated in the choice of these coordinates by M. Gudermann of Cleves, who is the author of an "Outline of Analytic Spherics," which Mr. Graves has been unable to procure.

The President communicated a new demonstration of Fourier's theorem.